An Improved Integral Model for a Non-Buoyant Turbulent Jet in Wave Environment

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Abstract: The integral model developed by Chin (1988) for modelling a non-buoyant turbulent jet in wave environment is improved by introducing two new parameters, i.e., the jet spreading rate $c_1$ and the shortening rate $p_e$. The parameter $c_1$ is used to simplify the model by explicitly describing the radial velocity and scalar profiles under the assumption of “instantaneous” Gaussian distribution. By doing so, the governing equations can be easily solved by simultaneously integrating the conservation laws of momentum and scalars across the jet cross-section. The parameter $p_e$ is used to shorten the initial length of zone of flow establishment (ZFE), so as to more accurately account for the wave effect on the jet initial dilution near the jet nozzle. The parameters are calibrated by the particle image velocimetry (PIV)-measured data from three groups of jet experiments, i.e., the group of vertical jet towards the wave direction (vertical jet), the group of horizontal jet along the wave direction (co-wave jet) and the group of horizontal jet opposing to the wave direction (op-wave jet). The results show that both parameters are well related to the ratio of jet and wave characteristic velocities in the same group, but it is not able to be generalized among different groups. Under the same wave condition, the value of $c_1$ in the vertical jet is larger than that of the horizontal jets; while the value of $p_e$ in the vertical jet is smaller than that of the horizontal jets, which indicates that the jet has a faster decay rate of centerline velocity and a wider width of jet cross-section profile in the near field when it is vertically discharged into the wave environment. With the well-calibrated parameters, the improved model can achieve a higher accuracy than the original model developed by Chin (1988).

Keywords: turbulent jet; regular waves; integral model; hydrodynamic characteristics; jet dilution

1. Introduction

Submarine outfalls play an important role in the disposal of treated wastewater to eliminate water pollution in the urban area of coastal cities. However, these outfalls are considered to be a kind of pollution source to the coastal waters, posing a high risk to the coastal and oceanic environment and ecological system [1–3]. In order to mitigate their influence, the outfall locations should be carefully determined so that the wastewater can be effectively diluted by the surrounding waters. To this end, it is essential to accurately predict the dilution processes of discharged wastewater in the coastal environment.

The movement of discharged wastewater from outfalls behaves typically like a turbulent jet. Under the effect of initial momentum and/or buoyancy, the surrounding waters are continuously entrained into the jet body, resulting in a significant reduction in the jet concentration during its initial mixing processes [4]. In coastal waters, due to the existence of waves, this mechanism becomes more
complicated. The experimental measurements by Chyan and Hwung [5] showed the existence of “twin peak” distribution of the jet mean velocity and concentration on the cross-sectional profiles when the jet is discharged into a regular wave environment. Mossa [6,7] measured both the mean velocity and the turbulent intensity using a Laser Doppler Anemometer (LDA) system and found a larger lateral spreading and a higher turbulence level of the jet in the wave environment than those in the stagnant ambience. Ryu et al. [8] measured the instantaneous velocity field using the particle image velocimetry (PIV) technique and revealed that the influence of wave amplitude on the rate of jet diffusion is significant while the influence of wave phase is relatively small. Chang et al. [9] found the wave-to-jet momentum ratio is the most important parameter to characterize the effect of waves on the jet initial dilution. Hsiao et al. [10] used a PIV technique to measure the mean and turbulence structure of a horizontal jet in the wave environment and obtained similar findings from the studies by Mossa [6,7] and Mossa and Davies [11]. Although the findings from the experimental studies could provide a good physical insight into the jet initial mixing processes in the wave environment, it is difficult to obtain the entire mixing processes of jet due to the limitation of measurement techniques as well as the high experimental costs.

The numerical model provides an alternative approach to investigate the jet in the wave environment. In literature, there are two main types of numerical models that can be used to study jet behaviors in the initial mixing zone. The first type is based on the solution of differential equations, which is costly in the computation, but has the ability to provide a detailed description of the jet mixing processes [12,13]. The second type is based on the solution of integral equations, which is efficient in the computation, but aims to describe the mean properties of the jet. In fact, the integral model can achieve fairly good results, while the computational costs are only with the order of minutes. That is the reason why the integral approach is increasingly popular for research purposes [14–22].

The general concept of the integral model is to integrate the governing equations by introducing some essential assumptions, such as the Gaussian distribution of the profiles of mean axial velocity and mean concentration. The integral model can be well developed for the jet in stagnant water and the jet in crossflow. For the jet in wave environment, some more assumptions are necessary to be introduced as the wave motion poses an unsteady environment. To tackle this problem, Chin [23] introduced the assumption of “instantaneous steadiness” to overcome the difficulty in dealing with the unsteady state and then developed an integral model to simulate the behavior of a buoyant jet in the wave environment. With the assumptions of Gaussian distribution and entrainment closure, the governing equations are solved by integrating the conservation laws of mass, momentum and scalars across the jet cross-section. The model was validated by his previous experimental data [24]. As the governing equation was simplified by using the assumption that the vertical velocity is much less than the horizontal velocity in his model, which implies that his model is confined to some specific ambient conditions and further modifications are necessary to fit for more general conditions. Later, similar to that proposed by Chin [23], Koole and Swan [25] developed another integral model which is based on the time-averaging of lateral displacement of Gaussian profile, with the pace of wave periodic movement. Both the momentum integral model and the related empirical formulas did not account for the changing effect of “zone of flow establishment”, but there has been proof that the “zone of flow establishment” can be considerably shortened by the wave motion. This makes Koole and Swan’s [25] model less accurate. By introducing the dynamic pressure gradient term into the equation of motion, which is zero for the traditional jet theory, Lin and Hsu [26] developed a new integral model, in which the action of waves is incorporated into the equations of motion as an external force. The model can be used to predict the trajectory, velocity distribution and boundary thickness of a buoyant jet over an arbitrary lateral cross section. However, because only the opposing averaged wave momentum induced by the radiation stress over a wave period was taken into account, this model can only be applied for simulating the cases of jet opposing to the direction of the wave propagation.

This study aims to develop a more generic and accurate model to simulate the turbulent jet in wave environment. For simplicity, this study will focus on the non-buoyant jet so that the buoyancy
effect can be neglected. Two new parameters, i.e., the jet spreading rate $c_1$ and the shortening rate $p_e$ are introduced. The parameter $c_1$ is used to simplify the model by explicitly describing the radial velocity & scalar profiles under the assumption of “instantaneous” Gaussian distribution. By doing so, the governing equations can easily be solved by simultaneously integrating the conservation laws of momentum and scalars across the jet cross-section. The parameter $p_e$ is used to shorten the initial length of zone of flow establishment (ZFE), in order to accurately analyze the wave effect on the jet initial dilution near the jet nozzle. The presence of two parameters $c_1$ and $p_e$ in the model is the key innovation in the present study. The model could be easily extended to more complicated cases such as an inclined jet discharged into the wave and current environment.

2. Model Descriptions

2.1. Global and Local Coordinate Systems

The definition diagram for a non-buoyant jet discharged into wave environment is given in a global Cartesian coordinate system, as shown in Figure 1, in which $x$ represents the direction of wave propagation and $z$ represents the direction upward against gravity $\hat{g}$. The ambient velocity is given by $\vec{U}$; $\vec{V}_c$ is the centerline velocity of the jet; $\vec{v}_r$ is the centerline velocity of the jet relative to the ambient; $\delta$ is the angle of the jet axis with respect to the $x$ axis; $\gamma$ is the angle of the relative velocity with respect to the jet axis; and $b$ is the jet width. The model has a constant density $\rho_a$. The turbulent round jet with diameter $D$ is located at $(0, 0, h_0)$ where $h_0$ is the height above the $x-y$ plane. It is oriented with a vertical angle $\delta_0$ between the jet center line and the horizontal $x$ axis. The jet has an un-sheared efflux velocity $u_0$, an efflux density $\rho_0$, which is the same as $\rho_a$, and an initial concentration $c_0$, representing the tracer or pollutant mass of interest. A local cylindrical coordinate system (Figure 1) with axial distance $s$, radial distance $r$ and azimuthal angle $\phi$ is defined along the trajectory.

![Figure 1. Definition diagram for a turbulent jet discharge into ambient flow with global and local coordinate system, respectively.](image)

The motion of wave-induced ambient fluid is given according to the linear wave theory [27],

\[
\begin{align*}
\bar{u} &= \frac{\pi H}{T} \frac{\cosh(kz)}{\sinh(kh)} \sin\left(kx - \frac{2\pi}{T}t\right) \\
\bar{w} &= \frac{\pi H}{T} \frac{\sinh(kz)}{\sinh(kh)} \cos\left(kx - \frac{2\pi}{T}t\right)
\end{align*}
\]
where $\bar{u}$ and $\bar{w}$ are the horizontal and vertical components of the wave induced velocity; $H$ is the wave height; $k$ is the wave number ($=2\pi/L$); $L$ is the wave length; $T$ is the wave period; $h$ is the water depth; $x$ and $z$ are the horizontal and vertical coordinates, respectively; and $t$ is the time.

2.2. Introduction of Chin’s [23] Model

2.2.1. Governing Equations

In Chin’s [23] model, the governing equations are given as follows,

Continuity equation:
$$ \frac{\partial}{\partial s} \int_0^\infty v \cos \gamma r dr = \alpha v_c b \cos \gamma + \frac{\beta v_c b \sin \gamma}{\pi} $$

$s$-momentum equation:
$$ \frac{\partial}{\partial s} \int_0^\infty v^2 \cos^2 \gamma r dr = g \sin \delta \int_0^\infty \frac{\Delta \rho}{\rho_0} r dr $$

$x$-momentum equation:
$$ \frac{\partial}{\partial s} \int_0^\infty v^2 \cos \gamma \cos(\delta + \gamma) r dr = 0 $$

Assume that $v \cos(\delta + \gamma) \approx v \cos \delta \cos \gamma$, the $x$-momentum equation can be simplified as,
$$ \frac{\partial}{\partial s} \int_0^\infty v^2 \cos^2 \gamma \cos \delta r dr = 0 $$

Density deficit equation:
$$ \frac{\partial}{\partial s} \int_0^\infty v \cos \gamma \Delta \rho r dr = 0 $$

Concentration equation:
$$ \frac{\partial}{\partial s} \int_0^\infty v \cos \gamma c r dr = 0 $$

where $\alpha$ is the radial entrainment coefficient; $\beta$ is the forced entrainment coefficient; $v$ is the velocity of the jet relative to the wave; $\Delta \rho$ is the density deficit between the jet and ambient flow; and $c$ is the concentration.

2.2.2. The Assumption of Radial Velocity and Scalar Profiles

In Chin’s [23] model, velocities, the radial profiles of velocity and scalar along the cross-sections are assumed to follow the Gaussian distribution,
$$ \begin{align*}
    v &= v_c \exp\left(-\frac{r^2}{b^2}\right) \\
    \Delta \rho &= \Delta \rho_c \exp\left(-\frac{r^2}{b^2}\right) \\
    c &= c_c \exp\left(-\frac{r^2}{b^2}\right)
\end{align*} $$

The subscript $c$ indicates the physical quantities of the jet element at the centerline.

2.2.3. Initial Conditions

The initial conditions for the solution of Chin’s [23] model are specified at the end of the ZFE. The non-dimensional length of the ZFE $s_{e*}$ is given as,
$$ s_{e*} = \begin{cases} 
    2.8F^{2/3} & F < 2 \\
    0.113F^2 + 4 & 2 \leq F \leq 3.2 \\
    5.6F^2 / (F^4 + 18)^{1/2} & F > 3.2
\end{cases} $$
where \( F = (u_0 - u_s \cos \delta_0) / ((\rho_a - \rho_0)gD / \rho_0)^{1/2} \), \( u_s \) is the wave-induced horizontal velocity near the jet nozzle. For the buoyant jet, as \( u_s \) varies in a sinusoidal form, the actual length of the ZFE \( s_{e*} \) is subject to the changing wave conditions. For the non-buoyant jet, \( F \) approaches infinity and \( s_{e*} \) is a constant which is equal to 5.6. From the conservation relations between the initial top-hat and Gaussian profiles, the initial condition for the concentration is given as,

\[
    c_{e*} = \frac{\lambda^2 + 1}{2\lambda^2} \tag{11}
\]

in which \( \lambda \) is the spreading ratio of concentration to velocity. Its typical value is equal to 1.20.

2.3. Modification of Chin’s [23] Model

2.3.1. Simplification of the Model for the Non-Buoyant Jet

As the non-buoyant jet can be considered as a special type of buoyant jet, the Chin’s [23] model is still applicable, but with the density difference equal to zero everywhere. Therefore, Equation (4) can be simplified as,

\[
    \frac{\partial}{\partial s} \int_0^\infty \nu^2 \cos^2 \gamma r dr = 0 \tag{12}
\]

and Equation (7) can be neglected.

2.3.2. Modification of x-Momentum Equation

In order to make the model more generic, the hypothesis \( \nu \cos(\delta + \gamma) \approx \nu \cos \delta \cos \gamma \) is not used in this study. Thus the x-momentum equation in the modified model is using Equation (5) rather than Equation (6).

2.3.3. Modification of the Radial Profiles of Velocity and Scalar

The momentum integral model is developed based on the assumptions of radial profiles of velocity and scalar. However, a detailed description of the radial profiles of the jet is still missing. Chin [23] suggested an assumption of “instantaneous steadiness”, which means the “instantaneous” radial profiles of jet velocity and scalar will follow the Gaussian distribution along the jet cross-sections during its oscillation with the wave motion. We follow Chin’s [23] assumption, but introduce a parameter \( c_1 \) into the “instantaneous” Gaussian distribution along the jet lateral profile,

\[
    \begin{aligned}
        v &= v_c \exp\left(-\frac{r^2}{2c_1^2}\right) \\
        c &= c_e \exp\left(-\frac{r^2}{2\lambda^2c_1^2}\right)
    \end{aligned} \tag{13}
\]

By doing so, the governing equations can be easily solved by simultaneously integrating the conservation laws of momentum and scalars across the jet cross-section. How to determine the value of parameter \( c_1 \) will be discussed in the following sections.

2.3.4. Modification of the Length of the ZFE

The experimental measurements presented by Chyan and Hwung [5] and Koole and Swan [25] clearly showed that the length of the ZFE can be considerably shortened by the wave motion. Therefore, on the solution of the present model, the coefficient \( p_e \), which represents the ratio of the length of the ZFE shortened by the wave motion to \( s_{e*} \), is introduced. The non-dimensional length of the ZFE \( p_e \times s_{e*} \) is used in the solution of the model. For the non-buoyant jet in wave environment, the non-dimensional length of the ZFE is equal to \( p_e \times s_{e*} = 5.6p_e \). How to determine the value of parameter \( p_e \) will be discussed in the following sections.
2.4. Normalized Governing Equations

In order to make the model more generic, the governing equations are normalized by three dimensional parameters, that is: (1) the effluent uniform velocity $u_0$, (2) the diameter of jet outlet $D$, and (3) the effluent concentration $c_0$. For convenience of expression, the axial component of the relative velocity $v_c \cos \gamma$ is replaced by the symbol $u$ and the superscript, *, is taken to indicate the non-dimensional quantities. Hence the normalized variables can be expressed as follows:

\[
\begin{align*}
    u^* &= \frac{u}{u_0} c^* = \frac{v_c}{c_0} \\
    F_0 &= \frac{u_0}{\sqrt{gD}} \overline{u}^* = \frac{u}{u_0} \overline{w}^* = \frac{w}{w_0} \\
    s^* &= \frac{s}{D} x^* = \frac{x}{D} z^* = \frac{z}{D}
\end{align*}
\]

With the dimensionless quantities specified above, the normalized governing equations can be expressed as

\[
\begin{align*}
    2u^* c^2 s^2 (\cos \delta - \tan \gamma \sin \delta) \frac{du^*}{ds^*} &= -u^* c^2 s^2 \sin \delta \tan^2 \gamma \frac{dv^*}{ds^*} \\
    -u^* c^2 s^2 (\sin \delta + \tan \gamma \cos \delta) \frac{dv^*}{ds^*} &= 2u^* c^2 s^2 (\cos \delta - \tan \gamma \sin \delta) = 0
\end{align*}
\]

\[
\begin{align*}
    2u^* c^2 s^2 \frac{d\overline{u}^*}{ds^*} + 2 u^* c s^2 = 0 \\
    c s^2 \frac{d\overline{w}^*}{ds^*} + u^* s^2 \frac{d\overline{w}^*}{ds^*} + 2u^* c s^2 = 0
\end{align*}
\]

Based on the calculation of the jet trajectory, from the geometric relationships as shown in Figure 2, the normalized trajectory equations can be derived as follows:

\[
\begin{align*}
    \frac{dx^*}{ds^*} &= \frac{\overline{u}^*}{u^*} + \cos \delta - \sin \delta \tan \gamma \\
    \frac{dz^*}{ds^*} &= \frac{\overline{w}^*}{u^*} + \sin \delta + \cos \delta \tan \gamma
\end{align*}
\]

As shown in Figure 2, there exists a geometric relationship between angles $\delta$ and $\gamma$. Rearranging and normalizing the geometric relationship, it yields

\[
u^* \sin \gamma - (\overline{u}^* \sin \delta - \overline{w}^* \cos \delta) \cos \gamma = 0
\]

Figure 2. Geometric relationships between velocities.
The above equation can be derived as follows

\[
\sin \gamma \frac{du}{ds} + (u^* \cos \gamma + \tilde{u}^* \sin \delta \sin \gamma - \tilde{w}^* \cos \delta \sin \gamma) \frac{dv}{ds} - (u^* \cos \delta \cos \gamma + \tilde{w}^* \sin \delta \cos \gamma) \frac{dw}{ds} - \tilde{u}^* \frac{dv}{ds} \sin \delta \cos \gamma + \tilde{w}^* \frac{dw}{ds} \cos \delta \cos \gamma = 0
\]  

(21)

Thus, six ordinary differential equations, i.e., Equations (15)–(19) and (21), can be used to solve six variables: \(x^*, z^*, \gamma, \delta, u^*\) and \(c^*\).

2.5. Computational Setup and Solving Procedures

The Lagrangian approach is used to simulate the movement and dilution of jet elements. At \(t = 0\), the first jet element is located at the end of the ZFE with Gaussian distributions of relative velocity \(u_e^*\) and concentration \(c_e^*\). The variables \(\delta_e\) and \(\gamma_e\) are determined by the jet discharge angle, herein the subscript \(e\) indicates the variable values of the jet element at the end of the ZFE. As for the non-buoyant jet in the regular wave environment,

\[
u_e^* = 1 - \left(\tilde{u}^2 + \tilde{w}^2\right)^{1/2} \cos(\delta - \varphi)
\]  

(22)

where \(\varphi\) is the angle of the wave-induced velocity direction with respect to the horizontal direction.

Given increments along the instantaneous plume axis, \(\Delta s\), the six ordinary differential equations of \(\frac{du}{ds}, \frac{dv}{ds}, \frac{dw}{ds}, \frac{dx}{ds}\) and \(\frac{dz}{ds}\) can be solved simultaneously by using the fourth-order Runge-Kutta algorithm. The model contains two variable parameters \(c_1\) and \(p_e\), which should be calibrated by the field or experimental data. To fit for the convergence criteria, the Lagrangian time increment is given by \(\Delta t = \Delta s / u\), where \(\Delta s\) is the marching step. The comparative studies by using \(\Delta s = 0.1D\) and \(\Delta s = 0.01D\) have been carried out, and the results are not much different. To save the computational costs, \(\Delta s = 0.1D\) is used for the following studies. The solving procedure is repeated until the jet element reaches the axial distance \(s = 300D\), where the jet further dilution is very little.

3. Experimental Setup

In order to calibrate the parameters \(c_1\) and \(p_e\) in the modified model, a series of laboratory experiments on the hydrodynamic behaviors of non-buoyant jets in the wave environment, were conducted in a 46.0 m long, 0.5 m wide and 1.0 m deep wave flume at the laboratory of College of Harbor, Coastal and Offshore Engineering, Hohai University. A round acrylic pipe, with the diameter \(D\) of 0.01 m, was installed at the mid-section of the flume. The jet was discharged vertically or horizontally through the pipe, with the jet nozzle 0.20 m above the bottom. The jet source is supplied from a constant head tank above the wave flume with an adjustable valve to control the volume flow rate. The waves are generated by a piston-type paddle movement. After propagating through the test section, the wave energy was dissipated by a wave absorber installed at the end of the flume. The reflection coefficients under the present experimental wave conditions were less than 6%. The sketch of the experimental setup is shown in Figure 3.
1.2 s and 1.4 s were conducted in the experimental study. The letters VW(90°), WC(0°) and WO(180°) denote the discharge angles of jet relative to the propagating wave direction, numbers 1~10 denote 10 different jet & wave conditions. In addition, six experimental cases of jet in stagnant water environment, specified with two different jet initial velocities, i.e., 0.707 m/s and 0.884 m/s; two different wave heights, i.e., 20 mm and 40 mm; three different wave periods, i.e., 1.0 s, 1.2 s and 1.4 s were conducted in the experimental study. The letters VS(90°), SC(0°) and SO(180°) denote the discharge angles of jet relative to the propagating wave direction, numbers 0~10 denote 10 different jet & wave conditions. In addition, six experimental cases of jet in stagnant water environment, specified with two different jet initial velocities, i.e., 0.707 m/s and 0.884 m/s were conducted for comparison. The letters VS(90°), SC(0°) and SO(180°) denote the discharge angles of jet relative to the propagating wave direction, numbers 0~1 denote the difference of jet initial velocity. In the above experimental cases, the jet to characteristic wave velocity ratio $R_{jw} = u_0 / u_w$ and $R_{jwV} = u_0 / v_w$ varies between 16~60 and 30~90 for the vertical jet and the horizontal jet respectively. Herein $u_w$ and $v_w$ are defined as the maximum horizontal and vertical velocities at the jet exit.

The stagnant water depth in the flume was 0.6 m in all experiments. The Particle Image Velocimetry (PIV) system was used to measure the velocities of the jet along the transverse center plane of the flume. The PIV system used in this study includes a dual-head pulsed laser, laser light sheet optics (Beamtech Optronics Co., Ltd., Beijing, China), a CCD camera, and a synchronizer (TSI Inc., Shoreview, MN, USA). The dual-head pulsed laser is a Nd:YAG laser (Beamtech Optronics Co., Ltd, Beijing, China) that has a 15 Hz repetition rate and 380 mJ pulse maximum energy output. The PIV images were recorded using a 14-bit CCD camera with a 2048 × 2048 pixel resolution and 15 frames per second (fps) maximum framing rate. The sampling frequency was set as 7.25 Hz, the time interval was 1000 μs and the total sampling time was about 30 s. The field of view of PIV system was 38 cm × 31 cm. For data analysis, a commercial software package INSIGHT developed by TSI Inc. (Shoreview, MN, USA), was used. Since the tracer’s density was almost the same as that of the surrounding water, the effect of buoyancy was ignored for simplicity.

As shown in Table 1, 30 experimental cases of jet in the wave environment, specified with three different discharge angles, i.e., 90° (vertical jet), 0° (horizontal jet, along the wave direction) and 180° (horizontal jet, opposing to the wave direction); two different jet initial velocities, i.e., 0.707 m/s and 0.884 m/s; two different wave heights, i.e., 20 mm and 40 mm; three different wave periods, i.e., 1.0 s, 1.2 s and 1.4 s were conducted in the experimental study. The letters VW(90°), WC(0°) and WO(180°) denote the discharge angles of jet relative to the propagating wave direction, numbers 1~10 denote 10 different jet & wave conditions. In addition, six experimental cases of jet in stagnant water environment, specified with two different jet initial velocities, i.e., 0.707 m/s and 0.884 m/s were conducted for comparison. The letters VS(90°), SC(0°) and SO(180°) denote the discharge angles of jet relative to the propagating wave direction, numbers 0~1 denote the difference of jet initial velocity. In the above experimental cases, the jet to characteristic wave velocity ratio $R_{jw} = u_0 / u_w$ and $R_{jwV} = u_0 / v_w$ varies between 16~60 and 30~90 for the vertical jet and the horizontal jet respectively. Herein $u_w$ and $v_w$ are defined as the maximum horizontal and vertical velocities at the jet exit.
Table 1. List of experimental cases for the jet in stagnant water and wave environments.

<table>
<thead>
<tr>
<th>Case</th>
<th>Angle</th>
<th>Jet Initial Velocity, ( u_0 )/m/s</th>
<th>Wave Period, T/s</th>
<th>Wave Height, H/m</th>
<th>( R_{jw} )</th>
<th>( R_{jwV} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>VS0</td>
<td>90° (vertical)</td>
<td>0.707</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>VS1</td>
<td>90° (vertical)</td>
<td>0.884</td>
<td>1.0</td>
<td>0.020</td>
<td>47.9659</td>
<td>—</td>
</tr>
<tr>
<td>VW1</td>
<td>90° (vertical)</td>
<td>0.707</td>
<td>1.0</td>
<td>0.040</td>
<td>23.9829</td>
<td>—</td>
</tr>
<tr>
<td>VW2</td>
<td>90° (vertical)</td>
<td>0.707</td>
<td>1.2</td>
<td>0.020</td>
<td>32.8500</td>
<td>—</td>
</tr>
<tr>
<td>VW3</td>
<td>90° (vertical)</td>
<td>0.707</td>
<td>1.2</td>
<td>0.040</td>
<td>16.4250</td>
<td>—</td>
</tr>
<tr>
<td>VW4</td>
<td>90° (vertical)</td>
<td>0.884</td>
<td>1.0</td>
<td>0.020</td>
<td>59.9743</td>
<td>—</td>
</tr>
<tr>
<td>VW5</td>
<td>90° (vertical)</td>
<td>0.884</td>
<td>1.0</td>
<td>0.040</td>
<td>29.9872</td>
<td>—</td>
</tr>
<tr>
<td>VW6</td>
<td>90° (vertical)</td>
<td>0.884</td>
<td>1.2</td>
<td>0.020</td>
<td>41.0742</td>
<td>—</td>
</tr>
<tr>
<td>VW7</td>
<td>90° (vertical)</td>
<td>0.884</td>
<td>1.2</td>
<td>0.040</td>
<td>20.5371</td>
<td>—</td>
</tr>
<tr>
<td>VW8</td>
<td>90° (vertical)</td>
<td>0.884</td>
<td>1.4</td>
<td>0.020</td>
<td>33.5891</td>
<td>—</td>
</tr>
<tr>
<td>VW9</td>
<td>90° (vertical)</td>
<td>0.884</td>
<td>1.4</td>
<td>0.040</td>
<td>16.7946</td>
<td>—</td>
</tr>
<tr>
<td>VW10</td>
<td>90° (vertical)</td>
<td>0.884</td>
<td>1.4</td>
<td>0.040</td>
<td>59.9743</td>
<td>—</td>
</tr>
<tr>
<td>SC0</td>
<td>0° (horizontal)</td>
<td>0.707</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>SC1</td>
<td>0° (horizontal)</td>
<td>0.884</td>
<td>—</td>
<td>—</td>
<td>—</td>
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<td>71.1857</td>
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<tr>
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<td>35.5928</td>
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<td>0.020</td>
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<tr>
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<td>1.2</td>
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<td>0.020</td>
<td>89.0072</td>
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<td>0.040</td>
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<td>0.884</td>
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<td>0.020</td>
<td>77.2584</td>
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</tr>
<tr>
<td>WC8</td>
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<td>0.040</td>
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<td>1.4</td>
<td>0.040</td>
<td>38.6911</td>
<td>—</td>
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</table>

In this study, the PIV measurement data of 20 consecutive waves are used to describe the average condition of jet behaviors under wave environment. After averaging the data for 20 wave periods, the time averaged velocity field and concentration field of the jet are obtained. Figure 4 shows the values of the normalized mean axial velocity \( w_c \) of the 6 cases of the jet in stagnant water environment along the distance away from the nozzle. It can be seen that the measured data from the present study fits well with the classic relationships suggested by Albertson et al. [29]. It illustrates that the data measured using the PIV system should be reliable to be used to for the further study.
4. Results and Discussion

4.1. Model Calibration

For the model calibration, all the jet and wave conditions specified in the integral model are identical to the ones measured in the experiments. According to the numerical results, regardless of the angles of injection (taking VW9 and WC8 as the examples), the jet in the wave environment sways due to the wave effect, with a faster decay of the jet centerline concentration, as shown in Figures 5 and 6. These findings are consistent with the conclusions made by other researchers [5,30].

Figure 4. Values of $w_c/u_0$ along the jet mean axis.

Figure 5. Concentration distributions at different phase moments of the vertical jet (VW9) in the wave environment.
well with the experimental data. The results calculated from the original Chin’s [23] model (shown in red line) have quite large differences from the measured data. Although the change of length of the ZFE (identical to the length of the ZFE used in the modified integral model) may improve Chin’s [23] model behavior (shown in blue line), the modified integral model proposed in this study still shows superiority to Chin’s [23] model. Similar results are observed for other cases but are not shown here for the sake of brevity.

Figure 6. Concentration distributions at different phase moments of the horizontal jet (WC8) in the wave environment.

Figure 7 shows the comparison of the experimental and numerical (marked as NVW7–NVW10, NWC1–NWC4, NWO5 and NWO6) centerline profiles of mean axial velocity for 10 experimental cases (VW7–VW10, WC1–WC4, WO5 and WO6). It can be seen that, despite small differences at the initial stage, the numerical results from the modified integral model (shown in black line) generally agree well with the experimental data. The results calculated from the original Chin’s [23] model (shown in red line) have quite large differences from the measured data. Although the change of length of the ZFE (identical to the length of the ZFE used in the modified integral model) may improve Chin’s [23] model behavior (shown in blue line), the modified integral model proposed in this study still shows superiority to Chin’s [23] model. Similar results are observed for other cases but are not shown here for the sake of brevity.

Figure 7. Cont.
Figure 7. Comparison of jet axial velocity along the centerline for the experimental cases of (a) VW7 & NVW7, (b) VW8 & NVW8, (c) VW9 & NVW9, (d) VW10 & NVW10, (e) WC1 & NWC1, (f) WC2 & NWC2, (g) WC3 & NWC3, (h) WC4 & NWC4, (i) WO5 & NWO5, and (j) WO6 & NWO6.

Figure 8 shows the experimental and numerical lateral profiles of jet axial velocity at three different levels for the vertical jet (taking VW1, VW3, VW5 and VW7 for examples), the horizontal jet along the wave direction (taking WC6 and WC8 for examples) and the horizontal jet opposing to the wave direction (taking WO2, WO4, WO9 and WO10 for examples). It can be observed that the numerical results are in good agreement with the experimental data, which indicates the robustness and accuracy of the model we developed in this study.
Figure 8. Cont.
Figure 8. The time-averaged vertical velocity distributions for the experimental cases of (a) VW1, (b) VW3, (c) VW5 and (d) VW7; and the time-averaged horizontal velocity distributions for the experimental cases of (e) WC6, (f) WC8, (g) WO2, (h) WO4, (i) WO9, (j) WO10.

It should be noted that the parameters $c_1$ and $p_e$ for each case are different. They vary with the jet and wave conditions, and their values have been calibrated and shown in Table 2. It can be seen that the length of the ZFE is considerably shortened by the wave motion. The stronger the wave motion, the shorter the length of the ZFE. This observation is consistent with the findings from Koole and Swan [25].
Table 2. Calibrated values of $c_1$ and $p_e$ for the non-buoyant jet in wave environments.

<table>
<thead>
<tr>
<th>Case</th>
<th>Jet Initial Velocity, $u_0$/m/s</th>
<th>Wave Period, $T$/s</th>
<th>Wave Height, $H$/m</th>
<th>$R_{jw}(R_{jwV})$</th>
<th>$c_1$</th>
<th>$p_e$</th>
</tr>
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<tbody>
<tr>
<td>VW1</td>
<td>0.707</td>
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<td>47.9659</td>
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<td>0.86</td>
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<td>0.72</td>
</tr>
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<td>0.040</td>
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<td>0.105</td>
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In order to generalize the variations of $c_1$ and $p_e$ for the vertical jet, a dimensionless parameter $R_{jw}$ is introduced,

$$R_{jw} = \frac{u_0}{u_w} \tag{23}$$

which represents the ratio of the jet characteristic velocity $u_0$ (initial velocity at the vertical jet exit) and the wave characteristic velocity $u_w$. As the wave force near the vertical jet orifice (as shown in Figure 5) has a critical influence on the jet fluctuation, following Xu et al. [31], the maximum horizontal particle velocity at the jet exit position is used as the wave characteristic velocity. Following the small-amplitude wave theory, this velocity can be expressed as

$$u_w = \frac{\pi H \cosh(kh_0)}{T \sinh(kh)} \tag{24}$$

where $h$ is the stagnant water depth; $h_0$ is the height of jet nozzle above the bottom. According to the experimental settings, $h$ is equal to 0.6 m and $h_0$ is equal to 0.2 m.

However, for the horizontal jet, the wave-induced vertical motion plays a more important role near the nozzle (as shown in Figure 6), the jet-to-wave velocity ratio $R_{jwV}$ for the horizontal jet in co-wave and op-wave environment is introduced,

$$R_{jwV} = \frac{u_0}{v_w} \tag{25}$$
where \( v_w \) is the maximum wave-induced vertical velocity at the jet exit position. It can be expressed as,

\[
v_w = \frac{\pi H \sinh(kh_0)}{T \sinh(kh)}
\]  

(26)

For illustration, the values of \( p_e \) for the vertical jet, the horizontal jet (co-wave) and horizontal jet (op-wave) are plotted in Figure 9 against \( R_{jw} \) or \( R_{jwV} \). The result shows that \( p_e \) increases linearly with respect to \( R_{jw} \) or \( R_{jwV} \) and can be well expressed by the following regression equations,

\[
p_e = 0.004826R_{jw} + 0.6180 \quad 16 < R_{jw} < 60 \quad \text{for the vertical jet}
\]  

(27)

\[
p_e = 0.003446R_{jwV} + 0.6373 \quad 30 < R_{jwV} < 90 \quad \text{for the horizontal jet (co-wave)}
\]  

(28)

\[
p_e = 0.002404R_{jwV} + 0.7992 \quad 30 < R_{jwV} < 90 \quad \text{for the horizontal jet (op-wave)}
\]  

(29)

These equations clearly show that under the same jet initial velocity, with an increase in wave height and wave period, the length of the ZFE becomes shorter; while under the same wave conditions, with an increase in the jet initial velocity, the length of the ZFE increases.

![Figure 9](image)

**Figure 9.** The regression equations of \( p_e \) versus \( R_{jw} \) or \( R_{jwV} \) for the vertical jet (shown in black line), the horizontal jet along the wave direction (shown in red line) and the horizontal jet opposing the wave direction (shown in blue line).

As for the parameter \( c_1 \), it is found to be reverse to change of \( R_{jw} \) or \( R_{jwV} \). The values of \( c_1 \) for the vertical jet, the horizontal jet (co-wave) and horizontal jet (op-wave) are plotted against \( R_{jw}^{-1}(R_{jwV}^{-1} = 1/R_{jw}) \) or \( R_{jwV}^{-1}(R_{jwV}^{-1} = 1/R_{jwV}) \) in Figure 10. It shows that \( c_1 \) increases linearly with respect to \( R_{jw}^{-1} \) or \( R_{jwV}^{-1} \) and can be well expressed by the following regression equations,

\[
c_1 = 0.6161R_{jw}^{-1} + 0.09994 \quad 16 < R_{jw} < 60 \quad \text{for the vertical jet}
\]  

(30)

\[
c_1 = 1.587R_{jwV}^{-1} + 0.07444 \quad 30 < R_{jwV} < 90 \quad \text{for the horizontal jet (co-wave)}
\]  

(31)

\[
c_1 = 1.045R_{jwV}^{-1} + 0.07789 \quad 30 < R_{jwV} < 90 \quad \text{for the horizontal jet (op-wave)}
\]  

(32)

These equations clearly show that under the same jet initial velocity, with an increase in wave height and wave period, the spreading rate \( c_1 \) becomes larger; while under the same wave conditions, with an increase in the jet initial velocity, the spreading rate \( c_1 \) becomes smaller.
Figure 11. Comparison of (a) $p_e$ and (b) $c_1$ for the vertical jet and horizontal jets in the same wave environments.

Figure 10. The regression equations of $c_1$ versus $R_{jw}^{-1}$ or $R_{jwV}^{-1}$ for the vertical jet (shown in black line), the horizontal jet along the wave direction (shown in red line) and the horizontal jet opposing the wave direction (shown in blue line).

4.2. Comparative Study of Vertical and Horizontal Round Jets in Wave Environment

Figure 11a,b shows the comparison of $p_e$ and $c_1$ for the jets with different angles in the same wave environment. It can be seen from Figure 11a that the length of the ZFE of the vertical jet is smaller than that of the horizontal jet in the co-wave environment, and the length of the ZFE of the horizontal jet in the co-wave environment is smaller than that of the horizontal jet in the op-wave environment. As the length of the ZFE determines the position at which the velocity starts to decay, if the length of the ZFE is shorter, the jet centerline velocity starts to decay earlier and thus decays faster. It can be seen from Figure 11b that the jet spreading rate of the vertical jet is greater than that of the horizontal jet in the co-wave environment; the jet spreading rate of the horizontal jet in the co-wave environment is larger than that of the horizontal jet in the op-wave environment. Therefore, the radial velocity profiles of the vertical jet is flatter (or wider) than those of the horizontal jets at the same position away from the jet nozzle; the radial velocity profiles of the horizontal jet in the co-wave environment is flatter (or wider) than those of the horizontal jet in the op-wave environment at the same position from the nozzle.

Figure 12 shows the experimental time-averaged velocity vectors for VW1, WC1 and WO1 as shown in Table 1. The comparison highlights the differences between these three jet groups. Particularly, the figure shows that the velocity profiles of the vertical jet is flatter than those of the horizontal jet, the velocity profiles of the horizontal jet (co-wave) is flatter than those of the horizontal jet (opposing wave). The spatial distributions of velocity profile are consistent with the varying decay rate along the centerline for the vertical jet and horizontal jets, as it have been discussed above.
non-linear wave-current interaction may also alter the jet dilution processes in the near field. It should be addressed that this paper mainly focuses on the simulation of non-buoyant jet by using the improved integral model. Further study is necessary for the buoyant jet as the buoyancy may significantly change the jet centerline, particularly for the horizontal jets. This will be discussed in our next paper.

By quantitative comparison of $c_1$ and $p_e$ for the vertical and horizontal round jets in wave environment, it is found that the length of the ZFE of the vertical jet is shorter than those of the horizontal jets, while the spreading rate for the vertical jet is larger than those of the horizontal jets. This is mainly due to the fact that the wave action becomes stronger towards the free surface. It is also found that the velocity profile of the horizontal jet along the wave direction is slightly wider and flatter than that of the horizontal jet opposing to the wave direction, which indicates the non-linear wave-current interaction may also alter the jet dilution processes in the near field. It should be addressed that this paper mainly focuses on the simulation of non-buoyant jet by using the improved integral model. Further study is necessary for the buoyant jet as the buoyancy may significantly change the jet centerline, particularly for the horizontal jets. This will be discussed in our next paper.

**Author Contributions:** Conceptualization, Y.C.; methodology, S.F.; validation, formal analysis, data curation, S.F. and Y.C.; writing—original draft preparation, S.F.; writing—review and editing, Y.C., Z.X., E.O. and S.L.

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References


