Nonlinear Body Slip Angle Observer for Electric Vehicle Stability Control

Cong Geng* and Yoichi Hori°

This paper proposes a nonlinear observer for Body Slip Angle (θ) estimation, in which a nonlinear tire model is adopted for the observer design. A newly developed method to identify parameters of road surface friction coefficient (μ) is introduced into this observer, which makes the observer adaptive to road condition changing. Simulations and field tests are conducted, where the feasibility of μ identification and effectiveness of the observer are checked, especially for nonlinear cornering situations.

Keywords: Electric Vehicle, Stability Control, Body Slip Angle Estimation, Nonlinear Observer, Friction Coefficient Identification

1. INTRODUCTION

An important advantage of electric vehicles (EVs) has been recognized is that motor’s controllability can provide more flexible and novel ideas for vehicle stability control [1] [2] [3]. Body slip angle (θ) is an important value for such control strategies. However, as sensors to measure θ value are very expensive, it needs to estimate θ from only variables measurable. The most difficult for θ estimation is the nonlinear characteristics of a vehicle. The main nonlinearity comes from the tire force saturation decided by tire vertical load and road friction. Nonlinear force characteristic of tires and the uncertainty of road conditions make vehicle characteristics change greatly while cornering in nonlinear areas compared with that in linear areas. So the effective observers must consider tire nonlinearity and must be adaptive to road friction changing [4] [5]. This gives the great challenge to the design of θ observer.

To solve the problems, this paper proposes a nonlinear observer for θ estimation in which a nonlinear tire model is adopted. This nonlinear tire model has higher accuracy over linear tire models to describe the tire characteristics as tire slip angle becomes large in vehicle’s nonlinear cornering conditions. In addition, by making use of electric vehicles, another important merit that EV’s motor torque can be estimated accurately: the method to identify a road surface friction coefficient is introduced to the observer. The observer structure is shown as Figure 1. Simulations and field tests are conducted to check the observer in different cornering conditions. Analysis of simulation results and field tests data demonstrate the performance of the proposed nonlinear observer.

2. NONLINEAR θ OBSERVER DESIGN

2.1 θ Observer Structure and Description

The nonlinear observer structure is shown as Figure 2. The estimate of θ is computed as predicted value from states equation corrected by output feedback.

The observer’s state equation is:

$$\dot{x} = f(u, \hat{x}) - K(\hat{y} - y)$$

(1)

In the observer, $f(u, \hat{x})$ describes the state equation with nonlinear tire model. The state variables, input variables and output variables are:

$$x = \begin{bmatrix} \beta \\ \gamma \end{bmatrix}, u = \begin{bmatrix} \delta_f \\ N \end{bmatrix}, y = \begin{bmatrix} \gamma \\ a_y \end{bmatrix}$$

where $\delta_f$ denotes the steering angle of the front wheel, $a_y$ denotes vehicle lateral acceleration, $N$ is direct yaw moment caused by differential longitudinal forces among tires, and $K$ is the feedback matrix of the observer.

The observer’s output equation is:
where $v$ denotes the velocity of vehicle.

The following tire model is adopted in the observer to describe the nonlinear characteristics of tire lateral force [6]:

\[
F_{yi} = \frac{k_{yi}}{\alpha_{yi}} \tan^{-1}\left(\frac{\pi}{2\mu F_{zi}} \right) C_{zi} \alpha_i
\]

where $F_{yi}$ denotes tire lateral force, $\alpha_{yi}$ side slip angle of tires, $C_{zi}$ tire cornering stiffness, $F_{zi}$ wheel vertical load, $\mu$ road friction coefficient, $k_{yi}$ influence coefficient of tire longitudinal force, $i$ index of tires.

Compared with the linear tire model adopted in the previous $\beta$ observers studies [7], with the nonlinear function of $\tan^{-1}$ and the additional parameter $\mu$, this nonlinear model can describe the saturation characteristics of tire lateral force as tire slip angle gets large.

The dynamics of vehicle is described as:

\[
\begin{aligned}
\dot{\gamma} &= \gamma - \dot{\gamma} \\
\dot{\alpha}_y &= v(\dot{\beta} + \dot{\gamma})
\end{aligned}
\]

(2)

where $\gamma$ is distance between mass center and rear axle, $F_{xy}$ is longitudinal forces of front tires, $F_{yx}$ and $F_{yr}$ are lateral forces of front and rear tires which can be calculated according to the above nonlinear tire model.

Considering the kinematics relationship as Equation 2 and that $\delta_f$ value is relatively small in the vehicle’s high speed situations, the observer’s nonlinear states equations are derived as:

\[
\begin{aligned}
\dot{\beta} &= \frac{l_z}{m} (F_{xf} + F_{yr}) - \dot{\gamma} \\
\dot{\gamma} &= \frac{l_z}{m} (l_y F_{xf} - l_r F_{yr} + N)
\end{aligned}
\]

(5)

2.2 Observer Feedback Matrix Design

Till now, an observer model with nonlinear form is derived. However, the nonlinear observer’s design and application are relatively much difficult [7]. So this paper tries applying linear observer design method to solve the nonlinear problem. To do this, the observer model is changed into the form of an equivalent linear two freedom model by adopting the value of extended tire cornering power $c'_p$, which is defined as:

\[
c'_p = \frac{F_y}{\alpha}
\]

(6)

where $F_y$ is the tire lateral force and slip angle $\alpha$ is the tire slip angle at its operating point. $c'_p$ is updated according to the calculated $\alpha$ value and $F_y$ value from tire lateral force model as equation (3).

The design method of linear $\beta$ observer refers to paper...
By adopting tire cornering power $c'_p$, the nonlinear observer state equation (5) can be described as an equivalent linear state equation (7) at each operating point:

$$\dot{x} = Ax + Bu$$

(7)

where,

$$A = \frac{-\left(C_f' + C_r' + C_{f\delta} + C_{r\delta}\right)}{mv} - I, \quad B = \frac{C_f' + C_r'}{I_v}, \quad x = \begin{bmatrix} B \\ y \end{bmatrix}, \quad u = \begin{bmatrix} \delta_f \\ N \end{bmatrix}$$

where, $C_f' \sim C_r'$ are the extended cornering power values of tires.

The above equations have the same structures as the linear observer of reference paper [8]. So the same design method of gain matrix $K$ can also be adopted. According to paper [8], $K$ is selected as following, for high response and robustness purposes.

$$K = \begin{bmatrix} \frac{1}{I_v \left(C_f' + C_r'\right)} - \frac{1}{I_v \left(C_f' + C_r'\right)} \right] - I, \quad \frac{1}{v} \begin{bmatrix} \frac{1}{I_v \left(C_f' + C_r'\right)} - \frac{1}{I_v \left(C_f' + C_r'\right)} \right]$$

where, $\lambda_1$ and $\lambda_2$ are the assigned pole values of the observer.

### 2.3 Road Surface Friction Coefficient Identification

Evidently, the accuracy of the proposed observer is determined by the tire model. According to equation (3), there are two basic parameters in the tire model: tire cornering stiffness $C$ and road friction coefficient $\mu$. Cornering stiffness $C$ is the characteristic of tires and is consistent, generally. Friction coefficient $\mu$ is uncertain and may change largely with the change of road conditions.

There have been many methods for $\mu$ identification proposed in previous studies [4] [5] [9]. Paper [9] put forward an effective way which comes from one of the excellent features of EVs, i.e., motor torque can be known easily and precisely from motor current. Therefore, the motor’s driving-force can be estimated easily. And then, the road friction coefficient identification based on driving force and wheel slip ratio can be realized. This paper introduces this $\mu$ identification method into the $\theta$ observer to adapt the road condition changes (the principal structure is shown as Figure 1). To check the feasibility of the $\mu$ identification, off-line calculation is conducted using the data measured by the experimental EV as the vehicle acceleration running in dry asphalt. The results confirm the validity of the introduced method.

### 3. SIMULATION STUDIES

To test the effectiveness of the nonlinear observer, Simulations are conducted with some typical steering angle input modes. The results of the comparison with

![Figure 3: Results of $\mu$ identification](image-url)
previous linear observer are shown in Figures 4 and 5. When the lateral acceleration is small, which means the tire's lateral forces are in their linear working region, \( \tilde{\theta} \) estimate values of both the linear and nonlinear observers with relatively high accuracy. With the lateral acceleration getting larger, which means the tire's lateral forces are in their nonlinear region, \( \tilde{\theta} \) estimate values of linear observers deviate from the real values. Comparatively, the nonlinear observer still has enough accuracy in such situations.

Figures 6 and 7 show the simulation results when different cornering stiffness and road surface friction coefficient values are applied in \( \tilde{\theta} \) observer. The estimate \( \tilde{\theta} \) value in a nonlinear region is insensitive with the deviation of cornering stiffness \( c \), for the tire lateral force tends to be its saturation value when tire slip angle gets large, which is independent with \( c \) value. The robustness over tire cornering stiffness error is one of the advantages of nonlinear observer over the linear ones. The results also show the estimate \( \tilde{\theta} \) values are more sensitive to the road surface friction coefficient errors. This is because of the reason that tire lateral force saturation values are determined by road friction conditions. As tire forces approach the road friction limitation, the calculated tire forces with incorrect \( \mu \) value may be quite different from the real ones. These simulations confirm the importance of \( \mu \) identification to ensure the observer's accuracy.

4. EXPERIMENTS OF \( \mu \) OBSERVER

To check the proposed \( \mu \) observer, field tests are conducted in our experimental EV, UOT March II. UOT March II is equipped with acceleration sensor, gyro sensor and noncontact speed meter which enable us to measure real vehicle state values. Results of field tests shown in Figure 8 and Figure 9 demonstrate the effectiveness of the nonlinear observer in both the linear and nonlinear cornering situations of the experimental vehicle.

5. CONCLUDING REMARKS

In this paper, by adopting a nonlinear tire model, a nonlinear \( \tilde{\theta} \) observer is proposed, which is checked to
Figure 6: Simulation results with different stiffness values (vehicle accelerates with constant $\delta_f = 3^\circ$)

Figure 7: Simulation results as observer friction coefficient ($\mu_{ob}$) is different from real one ($\mu_v$)

Figure 8: Field test results of $\beta$ observer
Linear area (driver steering angle=90°, v=40 km/h)

Figure 9: Field test results of $\beta$ observer
Nonlinear area (driver steering angle=90°, v=60 km/h)
be effective by simulations and field tests, especially in nonlinear cornering region. This is because that the nonlinear tire model can describe the saturation characteristics of tire lateral force. Another advantage of the proposed observer is its robustness over tire cornering stiffness parameter errors. To make the observer adaptive to road friction condition changing, method to identify parameters of road surface friction coefficient (\(\mu\) value) is introduced into this observer. The test results demonstrate the validity of \(\mu\) identification method. The planning future works are the adequate experimental studies on the effectiveness of the observer in different road conditions.

REFERENCES


AUTHORS

Cong Geng received her M.S. degree in Vehicle Engineering from Jilin University of Technology in 1996 and joined the Department of Transportation and Traffic as an Assistant Professor and became a Lecturer in 1999. She has worked at Beijing Jiaotong University as a Lecturer since 2000. She is a Ph.D. student at the University of Tokyo, where she has studied since 2006. She is interested in vehicle dynamics analysis and control technology. She also studies the advanced motion control of electric vehicles. Phone: 81-3-5452-6289, Fax: 81-3-5452-6288, geng@horilab.iis.u-tokyo.ac.jp.

Yoichi Hori received his Ph.D. degree in Electrical Engineering from the University of Tokyo in 1983 and joined the Department of Electrical Engineering as a Research Associate. He later became a Professor in 2000. In 2002, he moved to the Institute of Industrial Science as a Professor of Information & Electronics Division. His research fields are control theory and its industrial application to motion control, mechatronics, robotics, the electric vehicle, etc. He is an IEEE Fellow and worked as Treasurer of IEEE Japan Council and Tokyo Section from 2001 to 2002. He is now an AdCom member of IEEE-IES. He was the Vice President of IEE-Japan IAS from 2004 to 2005. He has been the chairman of ECaSS Forum since 2005. He was the program chairperson of EVS-22 in 2006. Phone: 81-3-5452-6287, Fax: 81-3-5452-6288, hori@iis.u-tokyo.ac.jp.